Received: *June 1, 2001*, Accepted: *July 14, 2001*

Gaussian and mean field approximations for reduced 4D supersymmetric Yang-Mills integral

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Abstract: In this paper, we consider a reduced supersymmetric Yang-Mills integral with four supercharges by using a gaussian approximation scheme and its improved version. We calculate the free energy and the expectation values of Polyakov loop and Wilson loop operators by extending the method employed in the bosonic case in the previous paper. Our results nicely match to the exact and the numerical results obtained before. The loop amplitudes exhibit good scaling behaviors similarly as in the bosonic case. The 't Hooft like large N limit leads simple formulas for the case of the loop length smaller. Also, the Polyakov loop and the Wilson loop are computed for the case of the loop length sufficiently large, where we see that the behavior of the Wilson loop reproduces the result simulated for a few smaller values of N at least qualitatively.

Keywords: Nonperturbative Effects, Superstrings and Heterotic Strings, M(atrix) Theories, Matrix Models.

JHEP07(2001)014

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JHEP07(2001)014

1. Introduction

In this paper, we consider a supersymmetric Yang-Mills integral, which is the complete dimensional reduction of the $\mathcal{N} = 1$ supersymmetric Yang-Mills theory on the euclidean four-dimensions with the gauge group SU(N). The classical action is $S = S^{(B)} + S^{(F)}$, where[1]

$$S^{(B)} = -\frac{1}{4g^2}\,\mathrm{tr}\,[X_\mu, X_\nu]^2\,, \qquad S^{(F)} = -\frac{i}{g^2}\,\mathrm{tr}\,\bar{\psi}_\alpha(\Gamma_\mu)_{\alpha\beta}[X_\mu, \psi_\beta]\,. \tag{1.1}$$

The variables X_μ ($\mu = 1, \ldots, 4$) are $N \times N$ traceless hermitian matrices, and ψ_α, $\bar{\psi}_\alpha$ ($\alpha = 1, 2$) are Grassmann valued traceless $N \times N$ complex matrices. For the Gamma

[1] This fermionic action $S^{(F)}$ differs from the action defined in ref. [3, eq. (2.1)] by the factor i. Unfortunately, the proof of the positivity of the fermion determinant $\mathcal{M}$ in [3] has a minor error. The identity $\sigma_2\mathcal{M}\sigma_2 = \mathcal{M}^*$ does not hold because $A_{a\mu}$ defined in [3, appendix A] is not real. However, the outline of the proof is correct. In fact, starting with our action and repeating the argument presented there, we can prove the positivity. Also, in the $N = 2$ case we are able to check it by an explicit calculation of the fermion determinant, which gives the answer identical to eq. (16) in the first paper of ref. [2]. Eventually, since the authors in [3] calculate everything by using the hermitian matrix $\mathcal{D} = \mathcal{M}^\dagger\mathcal{M}$, the factor i does not contribute and their final results are correct.

matrices Γ_μ we take a representation $\Gamma_k = i\sigma_k$, $\Gamma_4 = \mathbf{1}_2$, with σ_k $(k = 1, 2, 3)$ being the Pauli matrices. The partition function is defined as

$$Z = \int (dX)(d\psi d\bar{\psi})\, e^{-S} \tag{1.2}$$

with the integration measures

$$(dX) = \prod_{\mu=1}^{4} \prod_{A=1}^{N^2-1} \frac{dX_\mu^A}{\sqrt{2\pi}}, \qquad (d\psi d\bar{\psi}) = \prod_{\alpha=1}^{2} \left(\prod_{A=1}^{N^2-1} d\psi_\alpha^A d\bar{\psi}_\alpha^A \right). \tag{1.3}$$

The variables are expanded by a basis T^A normalized by $\mathrm{tr}\,(T^A T^B) = \frac{1}{2}\delta^{AB}$.

The extended version of this model with sixteen supercharges has been proposed as a nonperturbative definition of IIB superstring theory [1]. It has no expansion parameters because the coupling constant g in the action can be absorbed away into an appropriate rescaling of the variables. So it is hard to analyze it, as usual, in the form of an expansion of some parameters. Under this situation, numerical calculations have been performed vigorously [2]–[6]. Analytic exact results for the partition function and the correlator of some operators have been obtained by using the cohomological field theory approach based on supersymmetry [7, 8, 9].[2]

In this paper, we perform analytic calculations expanding the methods of gaussian and mean field approximations, discussed in the bosonic case in ref. [17], into the supersymmetric case. In the bosonic case, the partition function and various correlators including Polyakov and Wilson loops have been computed. The results for the gaussian approximation fit better to the known results obtained numerically or analytically as increasing the number of the variables (the rank of the gauge group N or the dimension D). In particular, it can be seen that the results for the Polyakov and Wilson loops nicely reproduce the numerical results in ref. [3] for the region of the loop length L smaller. Also, we tried evaluating the asymptotic behavior of the loop amplitudes in the limit $L \to \infty$ by improving the method. Here, we perform a similar analysis for the supersymmetric case. For various supersymmetric quantum mechanical systems, the method of the gaussian approximation has been examined in refs. [18, 19], which suggests that it is necessary to formulate the gaussian action so as to respect supersymmetry. This treatment is crucial, in particular, for observing the dynamical supersymmetry breaking from the behavior of the free energy. This is a reason why the gaussian action is considered in the superfield formalism including auxiliary fields in refs. [18, 19]. We follow this lesson and consider the supersymmetric gaussian action. In our case ($D = 4$), however we can easily realize the supersymmetry without using auxiliary fields. The situation is special for the four dimensions, so we will have to introduce auxiliary fields in the case of $D = 6$ or 10 (with eight or sixteen supercharges) [20].

[2]For analytical results from other methods, see [10, 11, 12]. Also, for the cohomological field theory approach to quantum mechanical and field theoretical matrix models, see [13]–[16].

This paper is organized as follows. In the next section 2, we consider the gaussian approximation to the supersymmetric Yang-Mills integral by extending the method used in the bosonic case [17] so as to respect supersymmetry. We obtain the results for the partition function and the amplitudes of Polyakov and Wilson loops, which are reliable for N larger and for the loop length L smaller. In section 3, in order to get some insight for behaviors of the loop amplitudes for L large, we consider an improved version of the gaussian approximation. The last section 4 is devoted to summarize our results and mention possible future directions.

2. Gaussian approximation

We consider the gaussian approximation for the supersymmetric Yang-Mills integral (1.1). As discussed in refs. [18, 19], respecting supersymmetry is crucial to realize a reasonable approximation. We start with the following gaussian action: $S_0 = S_0^{(B)} + S_0^{(F)}$,

$$S_0^{(B)} = \frac{1}{\sigma^2}\mathrm{tr}\,(X_\mu)^2\,, \qquad S_0^{(F)} = \frac{1}{\tau^2}\varepsilon_{\alpha\beta}\,\mathrm{tr}\,(\psi_\alpha\psi_\beta - \bar{\psi}_\alpha\bar{\psi}_\beta)\,. \tag{2.1}$$

This is supersymmetric, and the bosonic degrees of freedom and the fermionic ones are balanced. In fact, it is easy to see that the expectation value $\langle S_0\rangle_0$ under the classical action S_0 vanishes. In refs. [18, 19], for the gaussian approximation of supersymmetric quantum mechanical systems it is emphasized introducing auxiliary fields in order to balance the bosonic and fermionic degrees of freedom in gaussian actions. It is equivalent to matching the freedom in the off-shell level because in the gaussian action auxiliary fields contribute to the freedom as well as the dynamical variables. This is a reason why the gaussian action is considered starting with the superfield formalism in refs. [18, 19]. It is noted that in our zero-dimensional case the bosonic and fermionic degrees of freedom are already matched without introducing auxiliary fields. In this approximation, the partition function (the free energy F) and the correlator of the operator $\mathcal{A}$ are expanded as

$$F = F_0 - \sum_{n=1}^{\infty}\frac{(-1)^n}{n!}\langle (S-S_0)^n\rangle_{C,0}\,, \tag{2.2}$$

$$\langle\mathcal{A}\rangle = \langle\mathcal{A}\rangle_0 + \sum_{n=1}^{\infty}\frac{(-1)^n}{n!}\langle (S-S_0)^n\mathcal{A}\rangle_{C,0}\,, \tag{2.3}$$

where F_0 and $\langle\cdots\rangle_0$ denote the free energy and the expectation value in the gaussian theory. The suffix C means the connected part of the correlator.

2.1 Partition function

First we consider the partition function in the gaussian approximation. The outline is same as in the bosonic case [17], but in the supersymmetric case to get a reasonable result we have to reorganize the series of $S-S_0$ in (2.2) so that the expansion respects the supersymmetry. In order to do so, we introduce a number we will call "order" : 1 assigned to $S^{(B)}$, $S_0^{(B)}$ and $S_0^{(F)}$, and 1/2 to $S^{(F)}$. It is easy to see from a diagramatic consideration as for the gap equations in ref. [18] that the expansion with respect to the total number of the order is natural from the point of view of the supersymmetry of the gaussian action S_0. Straightforward calculations lead

$$\begin{aligned}
F_0 &= -2(N^2-1)\ln\frac{\sigma^2}{\tau^2} \\
\langle S^{(B)}-S_0\rangle_0 &= \frac{3\sigma^4}{2g^2}N(N^2-1) \\
-\frac{1}{2}\langle (S^{(F)})^2\rangle_{C,0} &= -\frac{1}{g^4}N(N^2-1)\sigma^2\tau^4 \\
-\frac{1}{2}\langle (S^{(B)}-S_0)^2\rangle_{C,0} &= -\frac{45\sigma^8}{16g^4}N^2(N^2-1)+\frac{3\sigma^4}{g^2}N(N^2-1) \\
\frac{1}{2}\langle (S^{(B)}-S_0)(S^{(F)})^2\rangle_{C,0} &= \frac{3\sigma^6\tau^4}{2g^6}N^2(N^2-1)-\frac{3\sigma^2\tau^4}{g^4}N(N^2-1) \\
-\frac{1}{4!}\langle (S^{(F)})^4\rangle_{C,0} &= \frac{5\sigma^4\tau^8}{8g^8}N^2(N^2-1)\,,
\end{aligned} \tag{2.4}$$

where the order 1 contributions are the second and third lines, and the order 2 contributions the forth and lowers. Considering the gap equations from the contribution up to the order 1, i.e.

$$\begin{aligned}
\frac{\partial}{\partial\sigma^2}\left[F_0+\langle S^{(B)}-S_0\rangle_0-\frac{1}{2}\langle (S^{(F)})^2\rangle_{C,0}\right] &= 0\,, \\
\frac{\partial}{\partial\tau^2}\left[F_0+\langle S^{(B)}-S_0\rangle_0-\frac{1}{2}\langle (S^{(F)})^2\rangle_{C,0}\right] &= 0\,,
\end{aligned} \tag{2.5}$$

the parameters are determined as

$$\sigma^2=\sqrt{\frac{g^2}{N}},\quad \tau^2=\left(\frac{g^6}{N}\right)^{1/4}. \tag{2.6}$$

Then the gaussian approximation for the free energy up to the order 2 is

$$F=\frac{1}{2}(N^2-1)\left[\ln(Ng^2)+1-\frac{11}{8}+(\text{corrections of (order)}\geq 3)\right], \tag{2.7}$$

where the second and third terms represent the corrections of the order 1 and 2, respectively.

Let us compare this result with the exact one [2, 7]. The exact result is given by

$$Z_{\text{exact}}=(2g^2)^{-\frac{1}{2}(N^2-1)}\mathcal{F}_N\frac{1}{N^2},\quad \mathcal{F}_N=\frac{2^{N(N+1)/2}\pi^{(N-1)/2}}{2\sqrt{N}\prod_{i=1}^{N-1}i!}, \tag{2.8}$$

and the large-N expanded form of the free energy becomes

$$F_{\rm exact} = -\ln Z_{\rm exact} = \frac{1}{2}(N^2-1)\left[\ln(Ng^2) - \frac{3}{2} + \frac{35}{6}\frac{\ln N}{N^2} + O(N^{-2})\right]. \qquad (2.9)$$

Taking the ratio $\Delta \equiv (F - F_{\rm exact})/F_{\rm exact}$ for various values of N, we have $\Delta(N=3) = -1.10408$, $\Delta(N=4) = -2.19021$, $\Delta(N=5) = -3.8156$, $\Delta(N=6) = -8.05504$, $\Delta(N=100) = 0.464781$, $\Delta(N=1000) = 0.23609$. Here we put $g^2 = 1/2$ for the same setting as in the bosonic case (in ref. [17, eq. (4.6)]). Note that differently from the bosonic case the inequality $F \leq F_0 + \langle S - S_0\rangle_0$ no longer holds because the integrals of Grassmann variables do not preserve the inequality. Thus Δ can take a negative value. Although for the smaller values of N (N = 3,4,5,6) the approximation does not make sense, for the larger values (N = 100, 1000) it becomes better as N increases. Also, for N larger it can be seen by comparing (2.7) with the expansion (2.9). On the other hand, in the bosonic case (see [17, eq. (4.6)]) the approximation gives a reasonable result already at the smaller N. We can say that in the supersymmetric case the convergence of the gaussian approximation for the partition function with respect to N is slower than that in the bosonic case.

2.2 Polyakov loop

Next we consider the expectation value of the Polyakov loop operator $\hat{P}(L) = \frac{1}{N}\mathrm{tr}\,(e^{iLX_1})$. In the expansion (2.3), calculation of the correlators $\langle \hat{P}(L)\rangle_0$, $-\langle \hat{P}(L)(S - S_0)\rangle_{C,0}$ and $\frac{1}{2}\langle \hat{P}(L)(S^{(B)} - S_0)^2\rangle_{C,0}$ reduces to that of the purely bosonic contributions, and leads to the same formulas as obtained in the bosonic case [17], due to decoupling of the fermionic contributions or to the parity symmetry $X_\mu \to -X_\mu$ of the gaussian action. So our task up to the order 2 reduces essentially to evaluating the three correlators $\frac{1}{2}\langle \hat{P}(L)(S^{(F)})^2\rangle_{C,0}$, $-\frac{1}{2}\langle \hat{P}(L)(S^{(B)} - S_0)(S^{(F)})^2\rangle_{C,0}$ and $\frac{1}{4!}\langle \hat{P}(L)(S^{(F)})^4\rangle_{C,0}$. In calculating the fermionic integrals, the formula

$$\langle \mathcal{B}(X)(S^{(F)})^2\rangle_{C,0} = \frac{\sigma^2\tau^4}{g^4}N\langle \mathcal{B}(X)S_0^{(B)}\rangle_{C,0} \qquad (2.10)$$

is useful. Here $\mathcal{B}(X)$ is a bosonic operator containing no ψ_α and $\bar{\psi}_\alpha$. The results are summarized as

- The lowest order

$$\langle \hat{P}(L)\rangle_0 = e^{-\frac{L^2\sigma^2}{4N}(N-1)}F\left(1-N, 2; \frac{L^2\sigma^2}{2}\right), \qquad (2.11)$$

- The order 1 correction

$$\begin{aligned} &-\langle \hat{P}(L)(S^{(B)} - S_0)\rangle_{C,0} + \frac{1}{2}\langle \hat{P}(L)(S^{(F)})^2\rangle_{C,0} = \\ &= \left(\frac{3\sigma^4}{2g^2}N - \frac{\sigma^2\tau^4}{2g^4}N - 1\right)\frac{L^2\sigma^2}{4}\left[e^{-\frac{L^2\sigma^2}{4N}(N-1)}\left\{(N+1)F\left(1-N, 3; \frac{L^2\sigma^2}{2}\right) - \right.\right. \\ &\qquad \left.\left. - F\left(1-N, 2; \frac{L^2\sigma^2}{2}\right)\right\} - \frac{1}{N}\langle \hat{P}(L)\rangle_0\right], \end{aligned}$$

- The order 2 correction

$$
\begin{aligned}
&\frac{1}{2}\langle \hat{P}(L)(S^{(B)}-S_0)^2\rangle_{C,0} - \frac{1}{2}\langle \hat{P}(L)(S^{(B)}-S_0)(S^{(F)})^2\rangle_{C,0} + \\
&\qquad\qquad + \frac{1}{4!}\langle \hat{P}(L)(S^{(F)})^4\rangle_{C,0} = \\
&= e^{-\frac{L^2\sigma^2}{4N}(N-1)}\left[-\left(\frac{3\sigma^4}{2g^2}N - \frac{\sigma^2\tau^4}{2g^4}N - 1\right)^2 + \frac{N-3}{4}\left(\frac{3\sigma^8}{g^4} - \frac{\sigma^4\tau^8}{2g^8}\right)\right]\times \\
&\quad \times \frac{L^4\sigma^4}{32N}\left[\binom{N}{4}F\left(4-N,5;\frac{L^2\sigma^2}{2}\right) + \binom{N+1}{4}F\left(3-N,5;\frac{L^2\sigma^2}{2}\right)\right] + \\
&\quad + e^{-\frac{L^2\sigma^2}{4N}(N-1)}\left[\left(\frac{3\sigma^4}{2g^2}N - \frac{\sigma^2\tau^4}{2g^4}N - 1\right)^2 + \frac{N+3}{4}\left(\frac{3\sigma^8}{g^4} - \frac{\sigma^4\tau^8}{2g^8}\right)\right]\times \\
&\quad \times \frac{L^4\sigma^4}{32N}\left[\binom{N+2}{4}F\left(2-N,5;\frac{L^2\sigma^2}{2}\right) + \right. \\
&\qquad\qquad \left. + \binom{N+3}{4}F\left(1-N,5;\frac{L^2\sigma^2}{2}\right)\right] + e^{-\frac{L^2\sigma^2}{4N}(N-1)}\times \\
&\quad \times \left(-\frac{3\sigma^8}{g^4} + \frac{\sigma^4\tau^8}{2g^8}\right)\frac{L^4\sigma^4}{32N}\left[\binom{N}{3}F\left(3-N,4;\frac{L^2\sigma^2}{2}\right) + \binom{N+1}{3}\times\right. \\
&\qquad\qquad\qquad\qquad \times F\left(2-N,4;\frac{L^2\sigma^2}{2}\right) + \binom{N+2}{3}\times \\
&\qquad\qquad\qquad\qquad \left.\times F\left(1-N,4;\frac{L^2\sigma^2}{2}\right)\right] + e^{-\frac{L^2\sigma^2}{4N}(N-1)}\times \\
&\quad \times \binom{N}{2}\left[\left\{-\left(\frac{9\sigma^4}{4g^2}N-1\right)^2 - \frac{3\sigma^8}{16g^4}N(3N-2) + \frac{\sigma^2\tau^4}{g^4}N\left(\frac{9\sigma^4}{4g^2}N-2\right) + \right.\right. \\
&\qquad\qquad \left.\left. + \frac{\sigma^4\tau^8}{16g^8}N(10N-1)\right\}\frac{L^2\sigma^2}{4N} - \left(\frac{3\sigma^4}{2g^2}N - \frac{\sigma^2\tau^4}{2g^4}N - 1\right)^2\frac{L^4\sigma^4}{16N^2}\right]\times \\
&\quad \times F\left(2-N,3;\frac{L^2\sigma^2}{2}\right) + e^{-\frac{L^2\sigma^2}{4N}(N-1)}\binom{N+1}{2}\times \\
&\quad \times \left[\left\{-\left(\frac{9\sigma^4}{4g^2}N-1\right)^2 - \frac{3\sigma^8}{16g^4}N(3N+2) + \frac{\sigma^2\tau^4}{g^4}N\left(\frac{9\sigma^4}{4g^2}N-2\right) + \right.\right. \\
&\qquad \left.\left. + \frac{\sigma^4\tau^8}{16g^8}N(10N+1)\right\}\frac{L^2\sigma^2}{4N} - \left(\frac{3\sigma^4}{2g^2}N - \frac{\sigma^2\tau^4}{2g^4}N - 1\right)^2\frac{L^4\sigma^4}{16N^2}\right]\times \\
&\qquad \times F\left(1-N,3;\frac{L^2\sigma^2}{2}\right) + \\
&\quad + \left[\left\{\left(\frac{9\sigma^4}{4g^2}N-1\right)^2 + \frac{15\sigma^8}{16g^4}N^2 - \frac{\sigma^2\tau^4}{g^4}N\left(\frac{9\sigma^4}{4g^2}N-2\right) - \frac{11\sigma^4\tau^8}{16g^8}N^2\right\}\times\right. \\
&\qquad \left.\times \frac{L^2\sigma^2}{4N} + \left(\frac{3\sigma^4}{2g^2}N - \frac{\sigma^2\tau^4}{2g^4}N - 1\right)^2\frac{L^4\sigma^4}{32N^2}\right]\langle\hat{P}(L)\rangle_0\,, \qquad (2.12)
\end{aligned}
$$

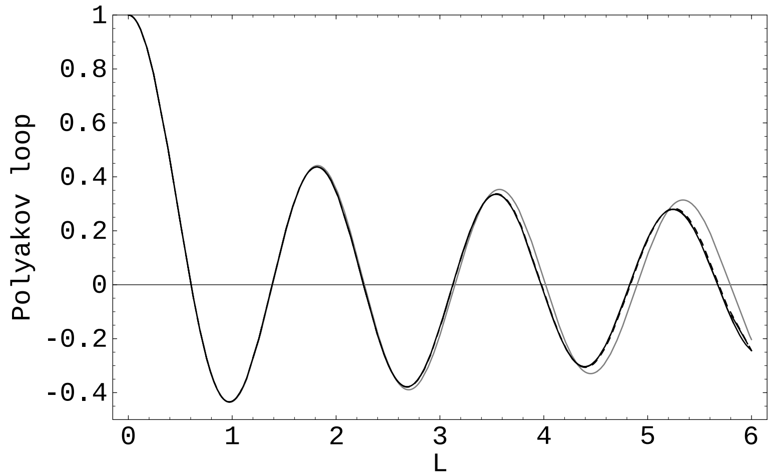

Figure 1: Polyakov loops up to the order 2 in the case $N = 16, 48, 100, 400$ and ∞. The gray line shows the case $N = 16$, the dotted line $N = 48$, the dashed line $N = 100$, the dot-dashed line $N = 400$, and the solid line $N = \infty$.

where F is the confluent hypergeometric function: $F(\alpha, \beta; z) = \sum_{n=0}^{\infty} \frac{\alpha(\alpha+1)\cdots(\alpha+n-1)}{\beta(\beta+1)\cdots(\beta+n-1)} \frac{z^n}{n!}$. Let us consider the value at the solution (2.6). Then, the order 1 correction vanishes, which is the same situation as in the bosonic case. Also, the 't Hooft like limit $N \to \infty$ with $Ng^2 \equiv G^2$ fixed leads to the simple formula expressed by the Bessel functions:

$$\langle \hat{P}(L) \rangle = \frac{1}{\sqrt{\alpha}} J_1(2\sqrt{\alpha}) + \frac{5}{16}\left[J_4(2\sqrt{\alpha}) - 4J_2(2\sqrt{\alpha})\right] + (\text{corrections of (order)} \geq 3)\,, \tag{2.13}$$

where $\alpha = L^2G/2$. In ref. [3] some numerical results about Polyakov and Wilson loops are reported. We can compare our result with the numerical one presented there. Let us take $Ng^2 = 48$ and consider the quantity for various values of N keeping Ng^2 fixed, which gives the same setting as in ref. [3]. Figure 5 in ref. [3] is the result to be compared with ours[3]. The variable $k/\sqrt{g}$ in the horizontal axis there corresponds to L in our setting. We plot our results up to the order 2 for various values of N in figure 1.

The result exhibits a quite nice scaling behavior in the region $0 < L < 6$ in particular for $N \geq 48$ so that the curves lie almost on top of each other. In figure 2 we show that the approximation gets better as increasing the order from 1 to 2. Comparing the curves for the order 1 (the dotted line) and the order 2 (the solid line) with the numerical result (the gray dots), we see that the order 2 result nicely reproduces the numerical result for the region of L smaller (up to about 0.6).[4] For

[3] There is a typo in the horizontal axis in figure 5 in ref. [3]. The horizontal axis represents $k/\sqrt{g}$ not k^2/g. I thank authors of ref. [3] for informing me of this fact.

[4] The exact result of the Polyakov loop amplitude is non-analytic at $L = 0$ because its second

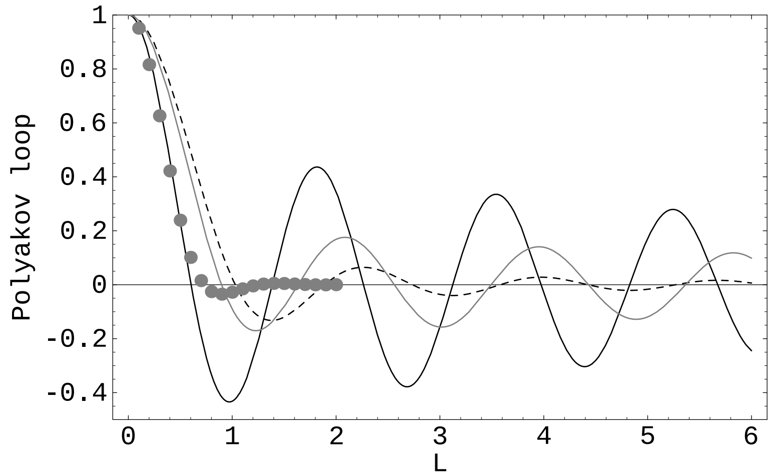

Figure 2: Polyakov loop in the case $N = \infty$ up to the order 1 (the dotted line) and up to the order 2 (the solid line). The gray dots indicate some points of the numerical data for the $N = 48$ case in ref. [3, figure 5]. The gray line is the result of the gaussian approximation (up to the second order) for the bosonic case $(D, N) = (4, \infty)$ in ref. [17, figure 3].

the region of L larger the contributions of the higher orders are considered to become more important. For the order 2 result in the region of L larger, the amplitude of the oscillation is larger compared to the bosonic case (the gray line). Agreement for the region seems to be not so better than that in the bosonic case as long as comparing the results up to the order 2.

2.3 Wilson loop

We calculate the expectation value of the Wilson loop $\hat{W}(L) = \frac{1}{N}\mathrm{tr}\,(e^{iLX_1}e^{iLX_2} \times e^{-iLX_1}e^{-iLX_2})$ by the gaussian approximation up to the order 1. The formulas for $\langle\hat{W}(L)\rangle_0$ and $-\langle\hat{W}(L)(S - S_0)\rangle_{C,0}$ are same as the ones obtained in the bosonic case [17] (eqs. from (4.28) to (4.37) in the paper). Thus the quantity we have to evaluate is only $\frac{1}{2}\langle\hat{W}(L)(S^{(F)})^2\rangle_{C,0}$, the result for which is

$$\frac{1}{2}\langle\hat{W}(L)(S^{(F)})^2\rangle_{C,0} = -\frac{\tau^4}{g^4}\frac{N^4}{N^2-1}\tilde{f}_2(L)(-1 + u(L))\,. \tag{2.14}$$

We used the functions $\tilde{f}_2(L)$ and $u(L)$ defined in ref. [17] (See eqs. (B.10) and (B.11) in the paper). The explicit formula is too lengthy to write down directly. However,

derivative $\langle\frac{1}{N}\mathrm{tr}(X_1)^2\rangle$ is logarithmically divergent [12]. The gaussian approximation up to the order 2 can not reproduce this non-analyticity, but the approximation is considered to get better in the 't Hooft limit from the observation that the non-analyticity vanishes in the 't Hooft limit [24].

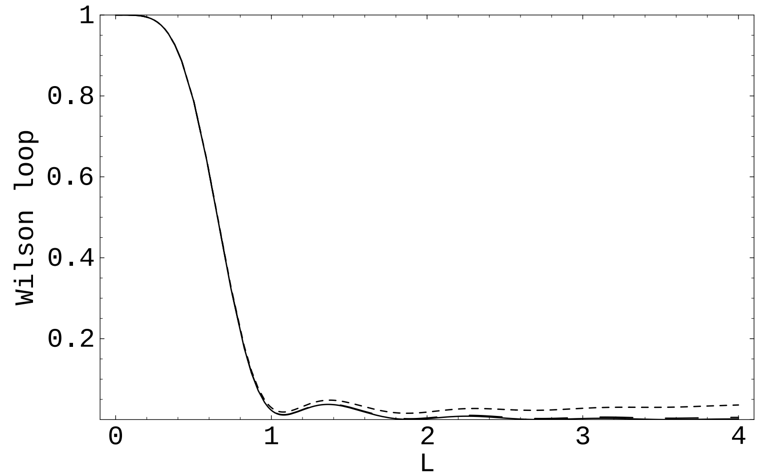

Figure 3: Wilson loops up to the order 1 for $N = 16$, 48 and ∞. The dotted line shows the case $N = 16$, the dashed line $N = 48$, and the solid line $N = \infty$. The scaling is so good that the curves for the $N = 48$ and ∞ cases are on top of each other.

in the 't Hooft like limit it becomes remarkably simplified as

$$\langle \hat{W}(L) \rangle = 1 - \left(\frac{1}{\alpha} (J_1(2\sqrt{\alpha}))^2 - 1 \right)^2 + \frac{1}{\alpha^2} \Big(9(J_1(2\sqrt{\alpha}))^2 (J_3(2\sqrt{\alpha}))^2 + 8(J_2(2\sqrt{\alpha}))^4 \Big) + \\ + (\text{corrections of (order)} \geq 2). \tag{2.15}$$

In figure 3, we plot the behavior for various values of N with $Ng^2 = 48$ being fixed. A nice scaling behavior for $N \geq 48$ can be seen as well as in the bosonic case [17].

Our result can be compared with the numerical result in ref. [3, figure 3]. Figure 4 shows our result with some data for $N = 48$ case in ref. [3, figure 3]. Taking into account the order 1 correction, the fitting of our result to the numerical one becomes better for the region of L smaller (up to about 0.7). Also, for the Wilson loop the behavior (the solid line) does not exhibit major discrepancies from that in the bosonic case (the gray line).

3. Improved mean field approximation

In order to obtain some insight for the behavior of the Polyakov and Wilson loops in the region of L large, we consider about the improved mean field approximation along the same line as discussed in the bosonic case [17]. First, we begin with obtaining an interpretation for the gaussian approximation as a mean field approximation. We take the following mean field action: $S_M = S_M^{(B)} + S_M^{(F)} + 1/\lambda$, where

$$S_M^{(B)} = \frac{1}{\sigma^2} \mathrm{tr}\, (X_\mu)^2 , \qquad S_M^{(F)} = \frac{1}{\tau^2} \varepsilon_{\alpha\beta} \mathrm{tr}\, (\psi_\alpha \psi_\beta) - \frac{1}{\bar{\tau}^2} \varepsilon_{\alpha\beta} \mathrm{tr}\, (\bar{\psi}_\alpha \bar{\psi}_\beta) ,$$

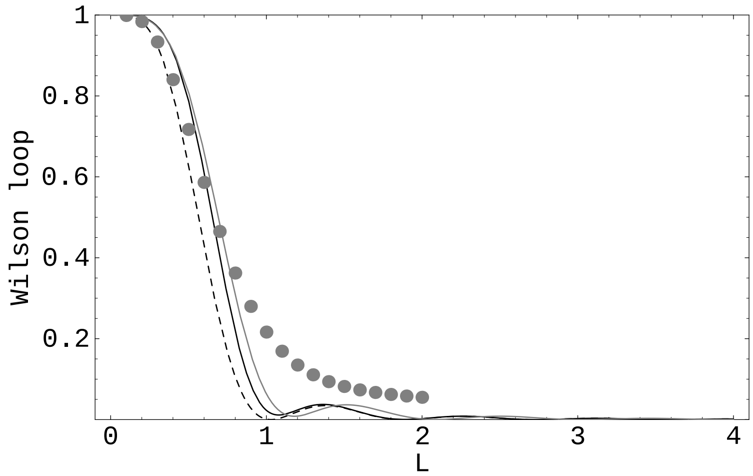

Figure 4: Wilson loop in the case $N = \infty$ up to the leading order (the dotted line) and up to the order 1 (the solid line). The gray dots indicate some points of the numerical result for $N = 48$ in ref. [3, figure 3]. The gray line, the result of the gaussian approximation (up to the first order) for the bosonic case $(D, N) = (4, \infty)$ in ref. [17, figure 6], is added for comparison.

$$\begin{aligned}
\frac{1}{\sigma^2} &= \frac{3}{4g^2}\frac{N^2}{N^2-1}\langle\frac{1}{N}\mathrm{tr}(X_\mu)^2\rangle + \frac{1}{2g^4}\frac{N}{(N^2-1)(N^2-2)}\varepsilon_{\alpha\beta}\varepsilon_{\gamma\delta}\times \\
&\quad\times\left[\langle\mathrm{tr}(\psi_\alpha\psi_\beta)\mathrm{tr}(\bar\psi_\gamma\bar\psi_\delta)\rangle + \langle\mathrm{tr}(\psi_\alpha\bar\psi_\gamma)\mathrm{tr}(\psi_\beta\bar\psi_\delta)\rangle\right], \\
\frac{1}{\tau^2} &= \frac{1}{2g^4}\frac{1}{(N^2-1)(N^2-2)}\varepsilon_{\alpha\beta}\left[\langle\mathrm{tr}(X_\mu)^2\mathrm{tr}(\bar\psi_\alpha\bar\psi_\beta)\rangle - \langle\mathrm{tr}(X_\mu\bar\psi_\alpha)\mathrm{tr}(X_\mu\bar\psi_\beta)\rangle\right], \\
\frac{1}{\bar\tau^2} &= -\frac{1}{2g^4}\frac{1}{(N^2-1)(N^2-2)}\varepsilon_{\alpha\beta}\left[\langle\mathrm{tr}(X_\mu)^2\mathrm{tr}(\psi_\alpha\psi_\beta)\rangle - \langle\mathrm{tr}(X_\mu\psi_\alpha)\mathrm{tr}(X_\mu\psi_\beta)\rangle\right], \quad (3.1)
\end{aligned}$$

λ is a constant determined later. The bosonic mean field action $S_M^{(B)}$ comes from the contraction of two bosons in $S^{(B)}$ and of four fermions in $\frac{1}{2}(S^{(F)})^2$. The fermionic action $S_M^{(F)}$ from the contraction of two bosons and two fermions in $\frac{1}{2}(S^{(F)})^2$. From the $O(4)$-rotational symmetry and the $\mathrm{SU}(N)$-gauge symmetry, the following index structures are assumed:

$$\begin{aligned}
\langle(X_\mu)_{ij}(X_\nu)_{kl}\rangle &= c\delta_{\mu\nu}\mathcal{I}_{ijkl}, && (3.2)\\
\langle(\psi_\alpha)_{ij}(\psi_\beta)_{kl}(\bar\psi_\gamma)_{mn}(\bar\psi_\delta)_{pq}\rangle &= \varepsilon_{\alpha\beta}\varepsilon_{\gamma\delta}\left[a\mathcal{I}_{ijkl}\mathcal{I}_{mnpq} + b\mathcal{I}_{ijmn}\mathcal{I}_{klpq} + b\mathcal{I}_{ijpq}\mathcal{I}_{klmn}\right], && (3.3)\\
\langle(X_\mu)_{ij}(X_\nu)_{kl}(\psi_\alpha)_{mn}(\psi_\beta)_{pq}\rangle &= \delta_{\mu\nu}\varepsilon_{\alpha\beta}\left[a'\mathcal{I}_{ijkl}\mathcal{I}_{mnpq} + b'\mathcal{I}_{ijmn}\mathcal{I}_{klpq} + b'\mathcal{I}_{ijpq}\mathcal{I}_{klmn}\right], && (3.4)
\end{aligned}$$

where c, a, b, a' and b' are scalar constants with no indices, and $\mathcal{I}_{ijkl} \equiv \delta_{il}\delta_{jk} - \frac{1}{N}\delta_{ij}\delta_{kl}$. We evaluate $\langle\cdots\rangle$ in eqs. (3.1) by replacing it with the expectation value under the mean field action $\langle\cdots\rangle_M$, leaving the self-consistency conditions

$$\frac{1}{\sigma^2} = \frac{3N}{2g^2}\sigma^2 - \frac{N}{2g^4}\tau^4, \quad \frac{1}{\tau^2} = \frac{N}{g^4}\sigma^2\tau^2. \quad (3.5)$$

Here we take τ to be real because we can always absorb the phases of τ and $\bar{\tau}$ by rotating the fermions in $S_M^{(F)}$ as $\psi \to e^{i\theta}\psi$, $\bar{\psi} \to e^{-i\theta}\bar{\psi}$. Eqs. (3.5) lead the solution (2.6). As same in the bosonic case [17], λ is determined by the requirement $Z = Z_M$ (Z_M is the partition function for the mean field action S_M.) as

$$\frac{1}{\lambda} = -\sum_{n=1}^{\infty} \frac{(-1)^n}{n!} \langle (S - S'_M)^n \rangle_{C,M} \,, \tag{3.6}$$

with $S'_M = S_M^{(B)} + S_M^{(F)}$. Then, the partition function becomes

$$F = -\ln Z = -\ln Z_M = -2(N^2-1)\ln\frac{\sigma^2}{\tau^2} - \sum_{n=1}^{\infty} \frac{(-1)^n}{n!} \langle (S - S'_M)^n \rangle_{C,M} \,, \tag{3.7}$$

that reproduces the gaussian approximation result in section 2.1.

Next, we consider the unnormalized correlator $\langle \mathcal{O} \rangle' \equiv \int (dX)(d\psi d\bar{\psi})\, \mathcal{O} e^{-S}$ where $\mathcal{O}$ is isotropic in the four dimensions and SU(N)-invariant. We repeat the above argument regarding $\langle \mathcal{O} \rangle'$ as the partition function of a theory with the Boltzmann weight $\mathcal{O}e^{-S}$. Then, for such $\mathcal{O}$, the correlators

$$\langle (X_\mu)_{ij}(X_\nu)_{kl}\mathcal{O} \rangle \,, \quad \langle (\psi_\alpha)_{ij}(\psi_\beta)_{kl}(\bar{\psi}_\gamma)_{mn}(\bar{\psi}_\delta)_{pq}\mathcal{O} \rangle \,, \quad \langle (X_\mu)_{ij}(X_\nu)_{kl}(\psi_\alpha)_{mn}(\psi_\beta)_{pq}\mathcal{O} \rangle$$

have the index structures same as the amplitudes without $\mathcal{O}$ in (3.2), (3.3) and (3.4), respectively. When starting with the mean field action: $S_M = S_M^{(B)} + S_M^{(F)} + 1/\tilde{\lambda}$,

$$S_M^{(B)} = \frac{1}{\tilde{\sigma}^2}\mathrm{tr}(X_\mu)^2 \,, \quad S_M^{(F)} = \frac{1}{\tilde{\tau}^2}\varepsilon_{\alpha\beta}\mathrm{tr}\psi_\alpha\psi_\beta - \frac{1}{\tilde{\bar{\tau}}^2}\varepsilon_{\alpha\beta}\mathrm{tr}\bar{\psi}_\alpha\bar{\psi}_\beta \,, \tag{3.8}$$

the self-consistency conditions give the formulas for $\tilde{\sigma}$, $\tilde{\tau}$ and $\tilde{\bar{\tau}}$ which are in the form same as eqs. (3.1) with the every correlators $\langle \cdots \rangle$ replaced to $\langle \cdots \rangle_M^{\mathcal{O}} \equiv \langle \cdots \mathcal{O} \rangle_M / \langle \mathcal{O} \rangle_M$. In particular, in the case that $\mathcal{O}$ depends only on X_μ's, which is a situation we will consider from now, these conditions read

$$\begin{aligned} \frac{1}{\tilde{\sigma}^2} &= \frac{3}{4g^2}\frac{N^2}{N^2-1}\langle \frac{1}{N}\mathrm{tr}(X_\mu)^2 \rangle_M^{\mathcal{O}} - \frac{N^2-1}{N}\frac{1}{\langle \frac{1}{N}\mathrm{tr}(X_\mu)^2 \rangle_M^{\mathcal{O}}} \,, \\ \frac{1}{\tilde{\tau}^4} &= \frac{1}{2g^4}\frac{N^2}{N^2-1}\langle \frac{1}{N}\mathrm{tr}(X_\mu)^2 \rangle_M^{\mathcal{O}} \,. \end{aligned} \tag{3.9}$$

Note that the computation of the quantity $\langle \frac{1}{N}\mathrm{tr}(X_\mu)^2 \rangle_M^{\mathcal{O}}$ completely reduces to the one in the bosonic case, because the fermionic contribution of the mean field action is factorized and cancelled in each expectation value $\langle \cdots \rangle_M$. From the requirement $\langle \mathcal{O} \rangle' = \langle \mathcal{O} \rangle'_M$, after the same calculation as in the bosonic case [17], $\tilde{\lambda}$ is determined as

$$\frac{1}{\tilde{\lambda}} = -\sum_{m=1}^{\infty} \frac{(-1)^m}{m!} \langle (S - S'_M)^m \rangle_{C,M} - \ln\left[1 + \frac{1}{\langle \mathcal{O} \rangle_M}\sum_{n=1}^{\infty} \frac{(-1)^n}{n!} \langle \mathcal{O}(S - S'_M)^n \rangle_{C,M}\right] . \tag{3.10}$$

The expression of the correlator becomes

$$\langle \mathcal{O} \rangle' = \left(\frac{\tilde{\sigma}^2}{\tilde{\tau}^2} \right)^{2(N^2-1)} e^{-1/\tilde{\lambda}} \langle \mathcal{O} \rangle_M \,. \tag{3.11}$$

Finally, dividing this by the partition function evaluated up to the order same as done for $\langle \mathcal{O} \rangle'$, the formula of the correlator $\langle \mathcal{O} \rangle$ is obtained. In the following two subsections, we try to evaluate the amplitudes of the Polyakov and Wilson loops for the case of L large.

3.1 Polyakov loop

For the Polyakov loop amplitude, we take the operator $\mathcal{O}$ as $\mathcal{O} = \frac{1}{2 \cdot 4} \frac{1}{N} \sum_{\mu=1}^{4} \mathrm{tr}(e^{iLX_\mu} + e^{-iLX_\mu})$. Obviously, if the rotational symmetry of $O(4)$ is not sponateously broken,[5] $\langle \mathcal{O} \rangle$ is equal to $\langle \hat{P}(L) \rangle$. When $x \equiv L^2 \tilde{\sigma}^2$ and $y \equiv L^3 \tilde{\tau}^2$ large, the consistency conditions (3.9) can be solved in the expanded form:

$$\begin{aligned} x_\pm &= e^{\pm i\pi/3} \left(\frac{16}{3}(N+1) L^4 g^2 \right)^{1/3} + \frac{4}{3} N(N+2) + O((L^4 g^2)^{-1/3}) \,, \\ y_\pm &= e^{\pm i\pi/6} (18(N+1))^{1/6} (L^4 g^2)^{2/3} \Bigg[1 + \frac{4}{3} N(4N+5) e^{\mp i\pi/3} \left(\frac{3}{16} \frac{1}{N+1} \frac{1}{L^4 g^2} \right)^{1/3} + \\ &\qquad + O((L^4 g^2)^{-2/3}) \Bigg]^{1/2} . \end{aligned} \tag{3.12}$$

We do not consider the solution such that $\mathrm{Re}\, x < 0$, because it leads to the unphysical result blowing up as $L \to \infty$. Corresponding to the above x, $\tilde{\lambda}$ is given by

$$\frac{1}{\tilde{\lambda}} = \begin{cases} (N-1) \left(\frac{1}{4N} x + N + O(x^{-1}) \right) & \text{(up to the order 1)} \\ (N-1) \left(\frac{1}{2N} x + 3N + 2 - \frac{5}{12} (N+1)^2 + O(x^{-1}) \right) & \text{(up to the order 2)} \,. \end{cases} \tag{3.13}$$

If we regard the solution as the sum of the $\pm$ branches similarly to the bosonic case [17], dividing the result corresponding to (3.11) by the partition function up to the order 1 or 2, we get the final answer

$$\begin{aligned} \langle \hat{P}(L) \rangle \sim C_{(n)} (L^4 g^2)^{-(N-1)^2/6} \exp \left[-u_{(n)} \frac{N-1}{N} \left(\frac{16}{3} (N+1) L^4 g^2 \right)^{1/3} \right] \times \\ \times \cos \left[\sqrt{3} u_{(n)} \frac{N-1}{N} \left(\frac{16}{3} (N+1) L^4 g^2 \right)^{1/3} - \frac{\pi}{3} (N-1)(N+2) \right] , \end{aligned} \tag{3.14}$$

[5] At least, in the case of N finite, this always holds. When it is spontaneously broken, if we regard the correlator $\langle \cdots \rangle$ as the sum of the expectation value at each superselection sector, the equality $\langle \mathcal{O} \rangle = \langle \hat{P}(L) \rangle$ holds. The spontaneous breaking of the rotational (Lorentz) symmetry in reduced Yang-Mills integrals is discussed in refs. [4],[21]–[25].

where up to the order 1 ($n = 1$): $u_{(1)} = 1/4$, and up to the order 2 ($n = 2$): $u_{(2)} = 3/8$. The overall constants are

$$
\begin{aligned}
C_{(1)} &= \frac{(-1)^{N-1} N^{(N^2-1)/2}}{2^{N-2} N!} (N+1)^{(N-1)(N+2)/3} \left(\frac{128}{81}\right)^{(N^2-1)/3} \left(\frac{16}{3}\right)^{(N-1)/3} \times \\
&\quad \times e^{-(N-1)(7N+5)/6}, \qquad (3.15) \\
C_{(2)} &= \frac{(-1)^{N-1} N^{(N^2-1)/2}}{2^{N-2} N!} (N+1)^{(N-1)(N+2)/3} \left(\frac{128}{81}\right)^{(N^2-1)/3} \left(\frac{16}{3}\right)^{(N-1)/3} \times \\
&\quad \times e^{(N^2-1)[\frac{5}{12}(N+1) - \frac{67}{16}]}. \qquad (3.16)
\end{aligned}
$$

It is interesting to compare this with the exact answer obtained in ref. [11]. In [11] the eigenvalue distribution $\rho(\lambda) = \langle \frac{1}{N} \mathrm{tr} \delta(\lambda - X_1) \rangle$ is exactly evaluated for the $N = 2$ case (see ref. [11, eq. (12)]). The Polyakov loop is related to this quantity via the Fourier transformation: $\langle \hat{P}(L) \rangle = \int_{-\infty}^{\infty} d\lambda\, e^{iL\lambda} \rho(\lambda)$. This integral can be easily estimated in the case L large, and we end up with

$$
\langle \hat{P}(L) \rangle = -\frac{\sqrt{3\pi}}{\Gamma(\frac{5}{2})} \left(\frac{L^4 g^2}{2^7}\right)^{1/3} \exp\left[-\frac{3}{2} \left(\frac{L^4 g^2}{2^7}\right)^{1/3}\right] \times [1 + O((L^4 g^2)^{-1/3})]. \qquad (3.17)
$$

Although our result reproduces the power $(L^4 g^2)^{1/3}$ of the exponential decay, the oscillating behavior can not be seen in the exact result, also the power behavior is different between the improved mean field result $((L^4 g^2)^{-1/6})$ and the exact result $((L^4 g^2)^{1/3})$. Moreover, the exponential decay behavior of the improved mean field result goes as $\exp\left[-2^{2/3} (2^{-7} L^4 g^2)^{1/3}\right]$ for up to the order 1 and $\exp[-3 \cdot 2^{-1/3} \times$ $\times (2^{-7} L^4 g^2)^{1/3}]$ for up to the order 2. Since the coefficients are $2^{2/3} = 1.58 \cdots$ and $3 \cdot 2^{-1/3} = 2.38 \cdots$, we can not say that the approximation tends to converge to the exact results as increasing the order. About this disagreement, we could consider as follows. The gaussian or mean field approximation becomes better when the number of variables is larger, which means that N is larger. As is seen in the analysis of the partition function in section 2.1, the result begins exhibiting the nice convergence for N larger than about 100. For smaller N ($N = O(1)$), the approximation does not make sense. There will be a possibility that the Polyakov loop has the same tendency as the partition function for the dependence of N and that our result makes sense for N large. In the result of the gaussian approximation in section 2.2, because all the confluent hypergeometric functions appearing there are polynomials of $L^2 \sigma^2$ for $N \geq 4$, the oscillating behavior terminates at some finite value of L. When N increases, the degree of the polynomials increases and the oscillating behavior continues to larger value of L. Thus it seems to be not ridiculous to expect that the asymptotic behavior of the Polyakov loop has the oscillating part in the case of N large.

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3.2 Wilson loop

We evaluate the Wilson loop amplitude up to the order 1. For the operator

$$\mathcal{O} = \frac{1}{4\cdot 3}\sum_{\mu\neq\nu}\frac{1}{N}\mathrm{tr}(e^{iLX_\mu}e^{iLX_\nu}e^{-iLX_\mu}e^{-iLX_\nu}),$$

we apply the procedure at the beginning of this section. The solution of the consistency condition (3.9) is given by

$$\begin{aligned}
\tilde{\sigma}^2 &= \sigma^2\left\{1-\frac{4}{3}\frac{1}{(N+1)!(N-1)!}e^{-\frac{L^2\sigma^2}{2}}\left(\frac{L^2\sigma^2}{2}\right)^{2N-2}+O\left(e^{-L^2\sigma^2}(L^2\sigma^2)^{4N-4}\right)\right\},\\
\tilde{\tau}^2 &= \tau^2\left\{1-\frac{1}{3}\frac{1}{(N+1)!(N-1)!}e^{-\frac{L^2\sigma^2}{2}}\left(\frac{L^2\sigma^2}{2}\right)^{2N-2}+O\left(e^{-L^2\sigma^2}(L^2\sigma^2)^{4N-4}\right)\right\},
\end{aligned}$$

with $\sigma^2 = \sqrt{\frac{g^2}{N}}$ and $\tau^2 = \left(\frac{g^6}{N}\right)^{1/4}$. In the limit $L\to\infty$, $\tilde{\sigma}$ and $\tilde{\tau}$ go to the values obtained by the gaussian approximation. $\tilde{\lambda}$ is determined as

$$\begin{aligned}
\frac{1}{\tilde{\lambda}} = \frac{1}{2}(N^2-1)\left(1-\frac{1}{N^2(N^2+3)}\right)+\frac{2}{N!(N-2)!}\times\\
\times\left\{-1+\frac{1}{N^2}\left[\frac{2}{3(N+3)}+2\frac{(N+1)(N+4)}{(N+2)(N+3)}-1\right]\right\}e^{-\frac{L^2\sigma^2}{2}}\left(\frac{L^2\sigma^2}{2}\right)^{2N-2}+\\
+O\left(e^{-\frac{L^2\sigma^2}{2}}(L^2\sigma^2)^{2N-3}\right).
\end{aligned}\tag{3.18}$$

After all, we obtain the final result up to the order 1

$$\begin{aligned}
\langle\hat{W}(L)\rangle = \frac{2}{N+1}\exp\left[\frac{N^2-1}{2N^2(N+3)}\right]\times\\
\times\left[1-\frac{1}{N!(N-2)!}\frac{2}{N^2}\left\{2\frac{(N+1)(N+4)}{(N+2)(N+3)}-1+\frac{2}{3(N+3)}\right\}\times\right.\\
\left.\times\, e^{-\frac{L^2\sigma^2}{2}}\left(\frac{L^2\sigma^2}{2}\right)^{2N-2}+O\left(e^{-\frac{L^2\sigma^2}{2}}(L^2\sigma^2)^{2N-3}\right)\right].
\end{aligned}\tag{3.19}$$

This behavior has the same tendency as in the bosonic case [17]. Namely, $\langle\hat{W}(L)\rangle$ has a nonzero limit as $L\to\infty$, and approaches to this limit from the below. The value of the limit $\frac{2}{N+1}\exp\left[\frac{N^2-1}{2N^2(N+3)}\right]$ becomes tiny as increasing N. It is consistent to the numerical result for L larger (up to $L=40$) and N smaller ($N = 2, 4, 8$) in ref. [11, figure 3(a)]. Since our approximation is expected to becomes better for N larger, we could say that for the Wilson loop our result nicely reproduces the true behavior at least qualitatively.

4. Discussions

We have considered the reduced four-dimensional supersymmetric Yang-Mills intergral with the four supercharges by using the gaussian and mean field approximations. We extended the approximation scheme considered in the bosonic Yang-Mills integrals so that the supersymmetry is respected. As a result, we obtained reasonable results for the partition function and the correlators of the Polyakov and Wilson loops. The result of the partition function shows a nice convergence into the exact value as N goes larger. Also the gaussian approximation for the Polyakov and Wilson loops fits well the numerical results for the region of the loop length L smaller. Next we considered the improved version of the mean field approximation in order to analyze the behavior of the loop amplitudes for L large. Although the result of the Polyakov loop does not exhibit good agreement to the exact result for the $N = 2$ case, it could be expected that the situation gets better for N large as we saw in the analysis of the partition function. The asymptotic behavior of the Wilson loop shows a nice behavior that reproduces the numerical result at least qualitatively.

Comparing with the situation in the bosonic case, we notice these two things. For both of bosonic and supersymmetric cases, as N goes larger, the result of the partition function tends to converge into the region where the approximation is reliable. For the supersymmetric case, however it is slower than that in the bosonic case. Also, in the gaussian approximation for the Polyakov loop, the difference for the region $L \geq 1$ between the approximate result and the numerical one is larger than in the bosonic case. Thus, we can conclude that as N goes smaller and L larger the gaussian approximation gets more worse comparing with the result in the bosonic case as long as looking at the first few orders. Since we have examined the first few terms in the expansion around the gaussian action, it is obvious to need investigate the higher order corrections. At present, however, computation up to the higher order gets much more complicated than that for the first few orders. We would need a calculation technique making the computation greatly simplified or an improvement of the approximation so as to give better results up to the first few orders.

An interesting direction is an investigation of the case of the supersymmetric Yang-Mills integral with eight or sixteen supercharges. In this case, there is a technical difficulty for obtaining reliable results by numerical simulations due to the fermion determinant being not positive definite.[6] Since our method seems to be extended also for this case by introducing auxiliary fields, it would be expected to give concrete results [20].

Acknowledgments

The authors would like to thank the authors of ref. [3], in particular W. Bietenholz and J. Nishimura for sending numerical data in the paper and correspondence.

[6] There are some attempts to find a way out of this problem in refs. [25].

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